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Excitation of the Magnetospheric Cavity by Space-Based ELF/VLF Transmitters

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14. ABSTRACT
During the period of performance Stanford University:
1. Developed an analytical model describing the distribution of current along a dipole antenna radiating ELF/VLF waves in the magnetospheric cavity. It was found that the antenna orientation had an important effect upon the radiation characteristics of the dipole antenna.
2. Mapped out the power distribution of ELF/VLF waves radiated by space-based transmitters throughout the magnetospheric cavity for a range of wave normal angles. It was found that the radiated waves can endure in the cavity for as long as 80 seconds before being absorbed through Landau damping.

15. SUBJECT TERMS
Space-based ELF/VLF transmitters, Magnetospheric cavity, ELF/VLF wave propagation

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1. SUMMARY

During the period of performance Stanford University developed an analytical model describing the distribution of current along a dipole antenna radiating ELF/VLF waves in the magnetospheric cavity. It was found that the antenna orientation had an important effect upon the radiation characteristics of the antenna. In addition, computer simulations were carried out concerning the storage within the magnetospheric cavity of ELF/VLF waves radiated by space-based ELF/VLF transmitters. It was found that these waves could be stored within the cavity for time periods of up to 80 seconds.

2. INTRODUCTION

The overall objectives of this research was to determine the: (1) optimum orbit for exciting the cavity resonance by a space-based ELF/VLF transmitter, (2) antenna type and configuration necessary to excite various cavity modes with the radiated ELF/VLF waves, (3) effects of Landau damping on the ELF/VLF waves within the cavity and examine possible methods of minimizing this damping, 4) effectiveness of the radiated ELF/VLF cavity waves in precipitating energetic radiation belt particles, and (5) optimum spacecraft orbit, antenna configuration, and ELF/VLF transmitter frequency spectrum for precipitating energetic radiation belt particles over a wide range of energies. The period of performance under this contract extended from October 31, 2003, through October 30, 2004.

3. RESULTS AND DISCUSSION

During the period of performance Stanford University: (1) Developed analytical models describing the current distribution of dipole antennas used to radiate ELF/VLF waves from spacecraft within the magnetosphere. (2) Carried out computer simulations describing the storage within the magnetospheric cavity of ELF/VLF waves radiated by space-based ELF/VLF transmitters.

3.1. Optimum Dipole Antenna Configuration

In the case of dipole antennas in the plasmasphere operating at ELF/VLF frequencies, it is not yet clear which orientations of the dipole are best suited to maximally excite the cavity resonance. In general at any given orientation, one would want to maximize the current along the dipole antenna in order to maximize the radiated power, while at the same time maximizing the wave energy radiated into the wave normal range of interest for cavity resonance excitation. During the reporting period, we have continued to investigate this question and have formulated integral equations describing the current along the antenna for the two cases in which the antenna is either parallel, or perpendicular, to the Earth's magnetic field, B_o, assumed to lie along the z axis. The integral equation for the case in which the antenna is parallel to B_o has the form:

$$\frac{\mu_o}{4\pi}\int_{-h}^{h}\frac{e^{-i\beta_m R_a}}{R_a}I(z^{'})dz^{'} = -i\frac{\sqrt{S}}{c}(b_o\cos\beta_s z + \frac{1}{2}V_o\sin\beta_s|z|) \tag{1}$$

where $R_a = [-\frac{|P|}{S}a^2 + (z-z^{'})^2]^{1/2}$, a is the antenna radius, h is the antenna half length, and b_o is a constant which is determined from the condition that the current, $I(z^{'})$, vanishes at the antenna end points, S is the dielectric constant along the x and y axes, P is the dielectric constant along the z axis, and $\beta_s^2 = S\omega^2/c^2$. To first order, the function $\frac{e^{-i\beta_m R_a}}{R_a}$ which appears in the integral shown in (1) has the characteristics of the delta function $\delta(z-z^{'})$. Thus, the current distribution is determined by the sinusoidal functions shown on the right hand side of (1). Surprisingly, in spite of the fact that the antenna current is flowing along the z axis, the wave length of the sinusoids describing the current depends upon the dielectric constant along the x and y axis, but not upon the dielectric constant along the z axis. This case illustrates the complex behavior of the plasma towards dipole antennas operating at ELF/VLF frequencies.

The integral equation for the case in which the antenna is perpendicular to B_o has the form:

$$\frac{\mu_o\gamma}{8\pi^2}\int_{0}^{2\pi}\int_{-h}^{h}\frac{e^{-\beta_c\gamma R_a}}{R_a}I(x^{'})dx^{'}d\phi = \frac{\beta_p}{\omega}(b_o\cosh\beta_p x - \frac{1}{2}V_o\sinh\beta_p|x|) \tag{2}$$

where $\beta_p = \sqrt{|P|}\omega/c$ and:

$$R_a = [(x-x^{'})^2 + (a\sin\phi)^2 - (a\cos\phi)^2/\gamma^2]^{1/2}$$

It can be seen that the right hand side of (2) consists of evanescent waves which decay exponentially along the antenna. Thus, to first order, the current will also decay exponentially along the antenna as $e^{-\beta_p x}$. In this case the current moment will increase only marginally if the antenna length is increased beyond the value $1/\beta_p$. We conclude that the orientation of the transmitting ELF/VLF antenna is an important component of a space-based ELF/VLF transmitting system. Further work on this topic will be carried out during the next reporting period.

3.2. Magnetospheric Cavity Resonance

ELF/VLF waves injected from a spacecraft in the inner magnetosphere distribute power throughout the radiation belts as a function of injection frequency and wave normal angle. Because the waves will undergo repeated magnetospheric reflections (MR), locations far from the injection point will be illuminated many times before the reflecting waves are attenuated through Landau damping. Landau damping occurs when the phase velocity of the wave is comparable to the rms velocity of the thermal electrons, and the particles absorb energy from the wave. As a result of the Landau damping, injected waves are slowly absorbed as they propagate, and their lifetimes vary from a few seconds to many tens of seconds. Combining a given raypath, as computed by the Stanford raytracing code, with a Landau damping calculation allows us to make a realistic estimate of how long the waves persist in the cavity and where the wave power is distributed.

For example, consider a 2.05 kHz wave injected at $L = 2$ from a spacecraft at the geomagnetic equator. As this wave propagates through the medium it will cross many magnetic L shells at various magnetic latitudes. Through the ray tracing code we can determine the wave power as a function of L value and magnetic latitude. To accomplish this for the wave in question we consider L-bins from L = 1.5 to L = 2.5, with a bin width of $\delta L = 0.05$. At each equatorial crossing we can make note of which L-bin the wave is in, as well as what fraction of the wave power remains after Landau damping. For each ray the wave power is initially normalized to the value 1.

Figure 1 shows the ray path, power, and L shell distribution of 2.05 kHz waves injected at the magnetic equator at $L = 2$ with 3 different wave normal angles, $\psi = -89°, -85°$, and $-75°$. The first column of panels in Figure 1 concerns the case $\psi = -89°$. This particular wave experiences more than 20 magnetospheric reflections and endures for more than 80 seconds before the wave power drops by 3 dB due to Landau damping. Thus, magnetospheric reflections allow a single wave to interact numerous times with the equatorial radiation belt electrons. The cavity enhancement factor plotted in the bottom panel of the first column is determined

by summing the normalized power at each equatorial crossing and dividing this power into the appropriate L-bins. The total cavity amplification factor is obtained by integrating the cavity enhancement factor across all L shells encountered by the wave. This allows us to quantify the cavity amplification factor. Note that the amplification effect can be greater than one despite starting with a normalized wave power of unity.

By varying the wave normal angle of injected waves at a given location, we can draw conclusions as to what types of waves will best illuminate different regions of the magnetosphere. For example, in the second column of Figure 1 it can be seen that a 2.05 kHz wave with an initial wave normal angle of ψ = 85° will propagate to higher L shells than the ray shown in the first column of Figure 1. However the wave endures for only 12 seconds before losing half of its power, and generally has a smaller cavity enhancement factor. Similarly, the ray shown is the third column of Figure 1 with the initial wave normal angle $\psi = -75°$ reaches higher L shells but endures for only 8 seconds before losing half of its power and has a generally smaller cavity enhancement factor than the first two rays. Further work on this topic will be carried out during the next reporting period.

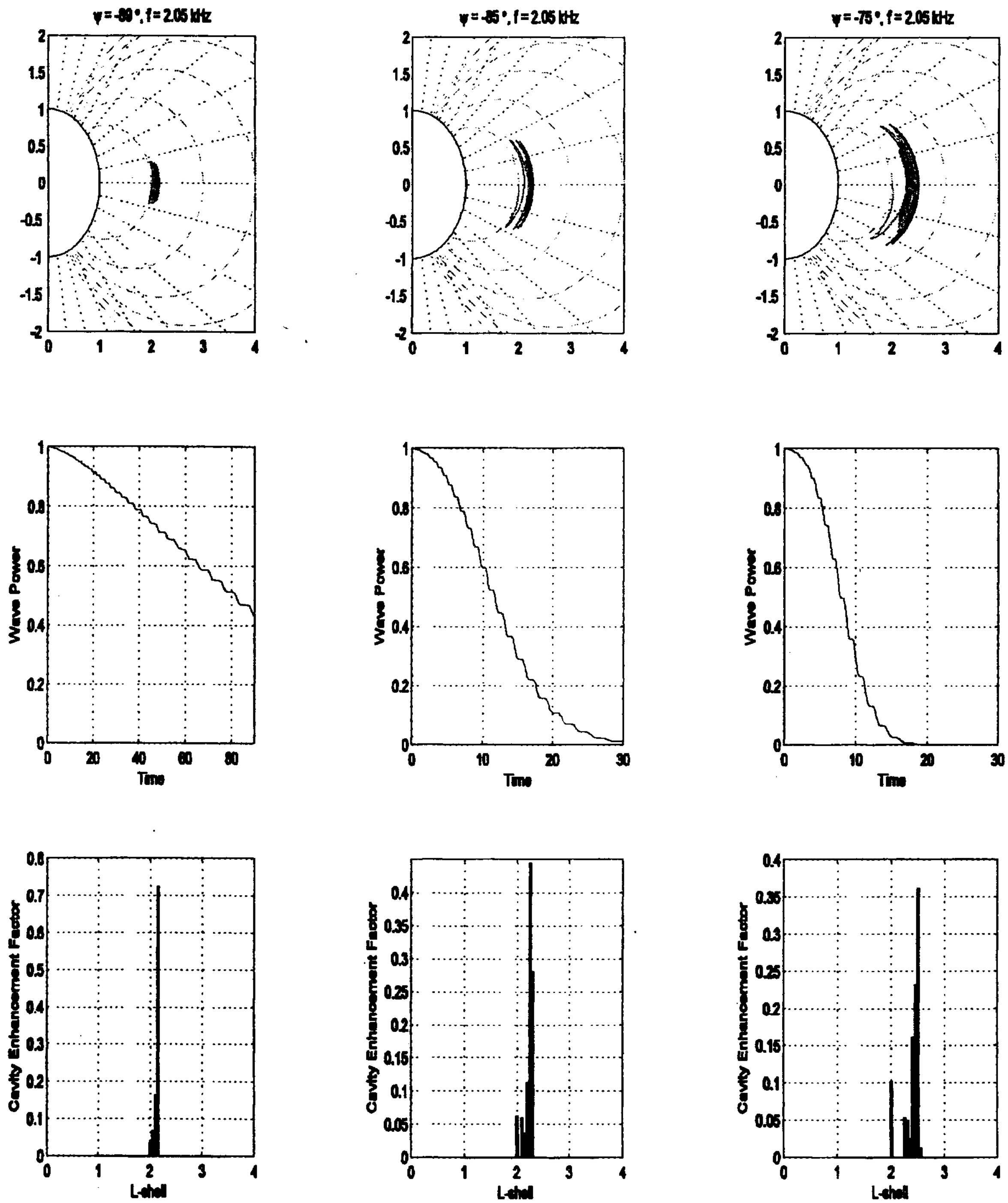

Figure 1. The ray paths, power, and L shell distribution of 2.05 kHz waves.